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ABSTRACT

A self consistent two dimensional, two component diffusion model is presented to describe the diurnal component of the thermosphere dynamics. In the regions where oxygen and helium are the minor constituents the diffusion process is shown to increase significantly the amplitudes of these constituents and to advance the times of their diurnal maxima by several hours with respect to the gas temperature and to the major species. Both effects are in basic agreement with satellite observations. The diffusion process is however relatively insignificant (< 10%) for temperature, total mass density and wind fields. The magnitude of the temperature-density phase delay is discussed in its dependence on the electron density distribution.

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INTRODUCTION

It has been shown in a preceding paper (Mayr and Volland, 1972a) that (a) the assumption of diffusive equilibrium is not valid for the diurnal variations in the thermosphere and (b) the diffusive redistribution of atomic oxygen in the two or three dimensional thermosphere dynamics could contribute to the observed temperature-density phase discrepancy. In these earlier calculations the wind and temperature fields were adopted from Volland and Mayr (1971) to generate the temperature and wind (diffusion) induced variations in atomic oxygen. Thereby the model was based on the assumption of a one way interaction in the sense that the drag feed back from the minor (O) to the major constituent (N_2) was neglected. Furthermore it was assumed that in the horizontal component the drag interaction between the thermospheric constituents strongly dominates the momentum transfer and thus no differentiation was made between the horizontal transport velocities of O and N_2.

Although it will be shown that none of these simplifying assumptions affect our earlier conclusions on the phase relations of the constituents, they imposed the restriction that the diffusion effects upon energetics and thermospheric circulation could not be treated. To overcome this deficiency, a two component model of the thermosphere dynamics has been developed.

Subject of this paper will be to describe the theory and the results that bear upon the phase relation between the gas temperature (Tg) and the concentrations of the constituents N_2, O and He.

THEORY

Considering a two dimensional model in which the Coriolis force is neglected, the equations of conservation of mass, momentum and energy are

$$\frac{\partial \rho_1}{\partial t} - Q_1 + L_1 + \frac{\partial}{\partial r}(\rho_1 W_1) + \frac{1}{r}\frac{\partial}{\partial \phi}(\rho_1 V_1) = 0 \qquad (1a)$$

$$\frac{\partial \rho_2}{\partial t} - Q_2 + L_2 + \frac{\partial}{\partial r}(\rho_2 W_2) + \frac{1}{r}\frac{\partial}{\partial \phi}(\rho_2 V_2) = 0 \qquad (1b)$$

$$\rho_1 \left\{ \frac{\partial V_1}{\partial t} + \nu_1 V_1 + \Phi \rho_2 (V_1 - V_2) \right\} - \eta_1 \frac{\partial^2 V_1}{\partial r^2} + \frac{1}{r}\frac{\partial p_1}{\partial \phi} = 0 \qquad (2a)$$

$$\rho_2 \left\{ \frac{\partial V_2}{\partial t} + \nu_2 V_2 + \Phi \rho_1 (V_2 - V_1) \right\} - \eta_2 \frac{\partial^2 V_2}{\partial r^2} + \frac{1}{r}\frac{\partial p_2}{\partial \phi} = 0 \qquad (2b)$$

$$\frac{\partial p_1}{\partial r} + \frac{(D m_1 + K \bar{m})}{m_1 (D + K)} g \rho_1 + \Phi \rho_1 \rho_2 (W_1 - W_2) = 0 \qquad (3a)$$

$$\frac{\partial p_2}{\partial r} + \frac{(D m_2 + K \bar{m})}{m_2 (D + K)} g \rho_2 + \Phi \rho_1 \rho_2 (W_2 - W_1) = 0 \qquad (3b)$$

$$\frac{3}{2} k \left(\frac{\rho_1}{m_1} + \frac{\rho_2}{m_2} \right) \frac{\partial T}{\partial t} - \frac{\partial}{\partial r}\left(\kappa \frac{\partial T}{\partial r} \right) + \rho_1 \frac{\partial W_1}{\partial r} + \rho_2 \frac{\partial W_2}{\partial r}$$

$$+ \frac{\rho_1}{r}\frac{\partial V_1}{\partial \phi} + \frac{\rho_2}{r}\frac{\partial V_2}{\partial \phi} = Q \qquad (4)$$

where the subscripts 1 and 2 refer to the two components (2 being the major one), and

ρ_1, ρ_2 = mass densities

W_1, W_2 = vertical transport velocities

V_1, V_2 = horizontal transport velocities

r_1, ϕ = radial distance, longitude

Q_1, Q_2 = chemical production terms

L_1, L_2 = chemical loss terms

ν_1, ν_2 = collision frequencies with ions

$$\Phi = \frac{k\,T}{(D + K)\, m_1\, \rho_2}$$

k = Boltzmann's constant

T = temperature

D, K = molecular, eddy diffusion coefficients

p_1, p_2 = pressures

η_1, η_2 = viscosity coefficients

κ = conductivity coefficient

Q = heat input rate

Although we have formally retained the photo production and loss rates for atomic oxygen these processes are of no significance for the diurnal variations since variations with this short period do not penetrate noticeably down to altitudes below 120 km.

Equations 1 through 4 are solved under the following assumptions:

(1) The variables are separated into time t, longitude ϕ and altitude r components in the form

$$\rho_{1,2} = \rho^0_{1,2} + \rho^1_{1,2}(r)\exp\{j\phi + j\omega(t - t_{\rho_{1,2}})\}$$

$$p_{1,2} = p^0_{1,2} + p^1_{1,2}(r)\exp\{j\phi + j\omega(t - t_{p_{1,2}})\}$$

$$T = T^0 + T^1(r)\exp\{j\phi + j\omega(t - t_{T_{1,2}})\} \qquad (5)$$

$$V_{1,2} = V^1_{1,2}\exp\{j\phi + j\omega(t - t_{V_{1,2}})\}$$

$$W_{1,2} = W^1_{1,2}\exp\{j\phi + j\omega(t - t_{W_{1,2}})\}$$

where ω is the angular frequency and $t_{i_{1,2}}$, are the phases (times the maxima) for the individual atmospheric parameters.

(2) Assuming that higher order terms in the frequency and longitude expansions can be neglected, perturbation theory is applied. This leads in a straight forward manner to a set of differential equations, not shown here in the interest of conciseness, which describes the height dependences in the amplitudes and phases of the various physical parameters.

The numerical scheme adopted to performing the integration of these equations is essentially that of Lindzen and Kuo (1969).

The boundary conditions employed in the calculations were the following:

(1) At the lower boundary we assumed that the characteristic time of the thermosphere is so long that no significant variations can be excited in the diurnal tide. Accordingly we assumed

$$\rho^1_{1,2} = p^1_{1,2} = W^1_{1,2} = V^1_{1,2} = T^1 = 0 \quad \text{at 90 km height} .$$

This assumption is of course somewhat arbitrary and therefore the solution is meaningless at the lower boundary level. However, it can be shown that the solution is very insensitive to this artificial constraint at higher altitudes (in our case above 120 km) where it can therefore be considered as unique.

(2) Due to the low internal energy and heat input rate it was assumed that at the upper boundary the transports of mass, energy and momentum have negligible effects upon the thermosphere dynamics. Accordingly we use

$$\frac{\partial V^1_{1,2}}{\partial r} = \frac{\partial T^1}{\partial r} = 0 \quad \text{at} \quad 500 \text{ km height.}$$

Again it must be noted that this assumption is strictly speaking not valid, with the effects from this ambiguity, however, being negligible below 400 km.

INPUT PARAMETERS

Following Volland and Mayr (1970) we adopted for the diurnal component of the heat input rate the height dependence

$$Q = Q_0 \exp\{-(r - r_0)/H\} \quad \text{for} \quad r \geq r_0 = 120 \text{ km}$$

with a scale height of H = 50 km. This distribution is very similar to the heat input determined by Harris and Priester (1962) for their CIRA 4-model (CIRA, 1965). A value of

$$Q_0 = 2 \times 10^{-7} \text{ erg/cm}^3 \text{ sec}$$

was used to match the observed diurnal variations of ±15% in the exospheric temperature. However, no significance should be attributed to this value of Q_0 since, in the framework of our perturbation theory, it does not affect the phases of the atmospheric parameters which will be primarily discussed here. Although the energy coupling from the lower atmosphere is relatively insignificant for the exospheric temperature variations (Volland and Mayr, 1972) it will still make some contribution and thus we are probably overestimating the internal heat input rate.

For the electron density, which enters into the ion collision frequencies, ν_1, ν_2 an altitude distribution of the form

$$N_e = N_m \exp\{(r - r_m)/H(r)\}$$

was used with H(r) varying in a form

$$H(r) = h_1 (r - r_m) + h_2 (r - r_m)^2,$$

such that the scale height becomes

$$H_1 = 80 \text{ km} \quad \text{at} \quad 120 \text{ km}$$

and

$$H_2 = -150 \text{ km} \quad \text{at} \quad 400 \text{ km}.$$

For the F_2 - maximum electron density a value of

$$N_m = 6 \times 10^5/cm^3$$

was adopted at $r_m = 300$ km. With the neutral-ion drag coefficient of

$$\xi = 7.5 \times 10^{-10}\ cm^3/sec \quad \text{(Dalgarno, 1964)}$$

the collision frequency $\nu \sim \nu_1 \sim \nu_2$ was derived from

$$\nu = \xi N_e \frac{\Omega^2}{\Omega^2 + \xi^2 n^2}$$

where $\Omega = 1.15 \times 10^2/sec$ is the Larmor frequency for ions and n is the time average number density of the neutral atmosphere.

The heat conductivity was computed from the function

$$\kappa = 5.25 \times 10^3 \frac{T^{1/2}}{m}$$

with m being the mean molecular mass in AMU. This form is in close agreement with the conductivity coefficients for the individual atmospheric constituents quoted by Harris and Priester (1962).

For the viscosity coefficient a form

$$\eta \sim \eta_1 \sim \eta_2 = \eta_0 T^{0\ 1/2}$$

was adopted, with a value of $\eta_0 = 1.3 \times 10^{-5}$ from Nicolet (1960).

Informations on the eddy diffusion coefficient are scarce, and since its value does not greatly influence the diurnal variations in the composition we have adopted a height independent value of

$$K = 4 \times 10^6\ cm^2/sec$$

from Colgrove et al. (1965).

For the molecular diffusion coefficient we used the function

$$D = D_0 \, (T/T_s)^{1.75} \, (p/p_s)^{-1}$$

where T_s and p_s are standard temperature and pressure, p being the sum of the partial pressures for the major constituents. From the work of Walker (1961) a value of

$$D_0 = 0.26 \text{ cm}^2/\text{sec}$$

was taken for the diffusion of oxygen through O_2 and N_2 . Considering that the diffusion coefficient depends on the square root of the reduced mass for the collision partners a value of

$$D_0 = 0.40 \text{ cm}^2/\text{sec}$$

was adopted for helium.

For the time average temperature distributions we adopted above 120 km a Jacchia-1965-model with T = 1050°K, while below 120 km the temperatures were taken from CIRA (1965). For the densities at 120 km the values were chosen from Jacchia (1965).

It was assumed that the time average densities are in diffusive equilibrium. However, deviating from Jacchia's model, we adopted a constant eddy diffusion coefficient over the entire altitude range and therefore the transition between mixing and diffusive separation is continuous.

In the continuity equations (1) through (4), r is related to the earth radius by

$$r = R \sin \theta$$

where θ is the co-latitude. In our analysis we have chosen $\theta = 45°$.

The model that is being discussed here is two dimensional in that we do not consider the effects from the latitudinal wind component. One should, however, expect (e.g. Volland and Mayr, 1972; Mayr and Volland, 1972a) that this meridional component significantly contributes to the energy and mass transport.

Thus we probably tend to underestimate these effects, and in our efforts to emphasize their importance we should therefore be on the conservative side.

DISCUSSION

Figure 1 shows a very simplified block diagram for the thermosphere dynamics. It illustrates the links between the composition and gas temperature on one hand and the various physical processes and atmospheric parameters on the other hand.

Suppose the heat input Q is known as a function of time, height, and latitude. Some of this energy input is conducted down into the lower atmosphere, thus affecting the temperature distribution and with that the composition. This aspect of the energetics has been the basis for Harris and Priester's (1962) one dimensional model of the diurnal variations in the thermosphere.

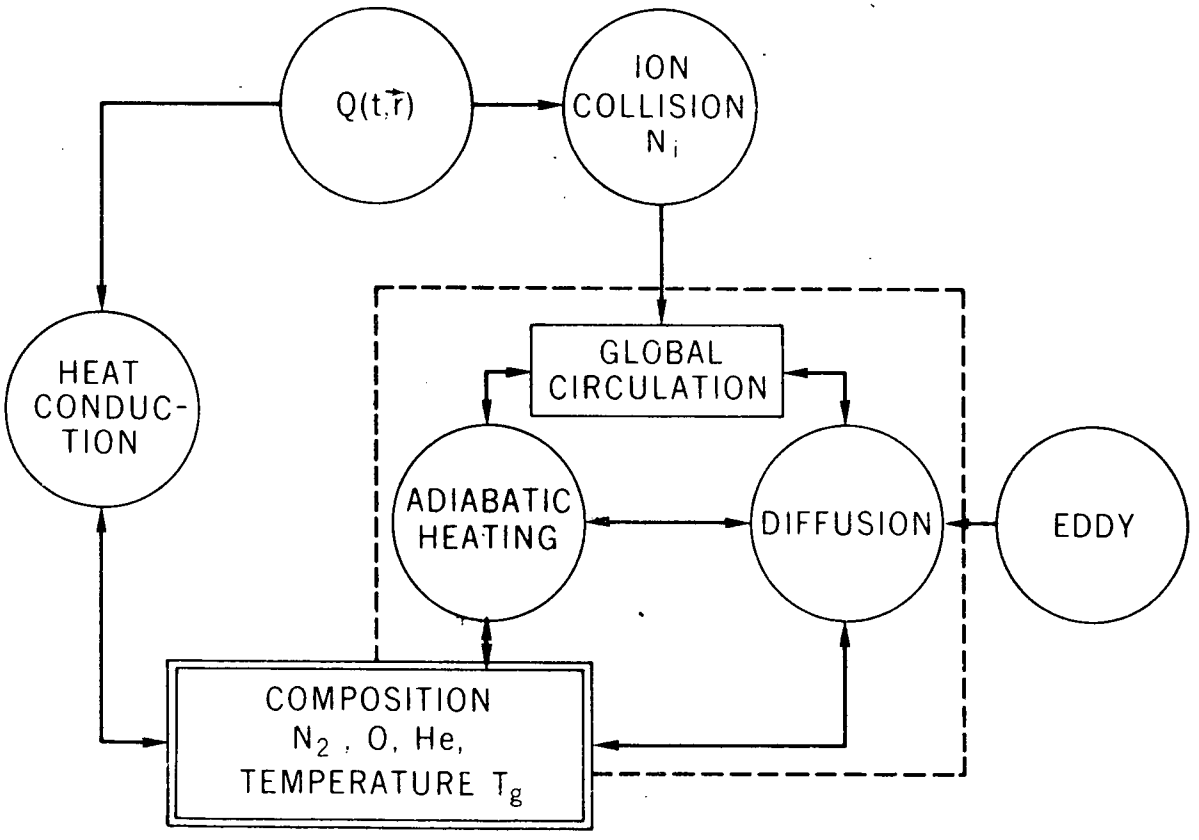

Figure 1. Schematic diagram for the physical processes that influence the temperature and composition of the thermosphere.

However, a second and not less important energy train goes through the global circulation (Dickinson et al., 1968; Volland and Mayr, 1970) which is significantly affected by ion collisions and thus by the ion density. In this mechanism, which is explicitly neglected in one dimensional models, energy is exchanged between day and night through adiabatic expansion and contraction and this in turn affects the temperature and the thermospheric composition.

In parallel, the thermospheric circulation induces diffusion which depends to various degrees upon the eddy diffusion coefficient. This diffusion process affects the composition directly. Any variations in the composition are associated with variations in the total pressure field. This in turn influences the global circulation and with that adiabatic heating and with that again temperature and composition. So it is apparent that all the processes that have been surrounded by the dashed box in Figure 1 are interconnected. In fact, it can be shown that the link between diffusion and energetics is extremely important particularly when we deal with magnetic storm and seasonal variations in the thermosphere (Mayr and Volland, 1972b).

PHASE RELATIONS

In Figure 2 we show the relative density and temperature variations as well as the phases (or times of maxima) for the diurnal component from our $O\text{-}N_2$ model. Two conditions are considered, one, in which we assume that the momentum transfer between O and N_2 is negligible corresponding to the assumption of diffusive equilibrium, ($\Phi = K = O$ in Eqs. (2) and (3)), and a second realistic one, in which we considered the diffusive interaction. It is apparent from this comparison that the diffusion process has a significant affect on the amplitudes and phases in the diurnal component of the composition, while its effect is negligible for the temperature variations.

In the "diffusive equilibrium solution" one sees the isopicnic levels for N_2 at about 160 and for O at 200 km with both constituents being in phase with the temperature above 180 km. In the "diffusion solution" the isopicnic level in N_2 is only slightly higher and this may reflect upon the difference in the time average thermospheric density distribution which is affected by the eddy diffusion coefficient below 120 km. Above about 180 km the diffusion effect on the amplitude of N_2 is negligible.

However, for O, our results show a number of rather significant deviations. The density amplitude up to 220 km is substantially higher in the "diffusion solution" while it is only slightly enhanced at higher altitudes. The most drastic effect becomes apparent in the height distribution of the oxygen phase which occurs significantly earlier than that of N_2 up to exospheric heights, basically in agreement with our earlier result (Mayr and Volland, 1972a).

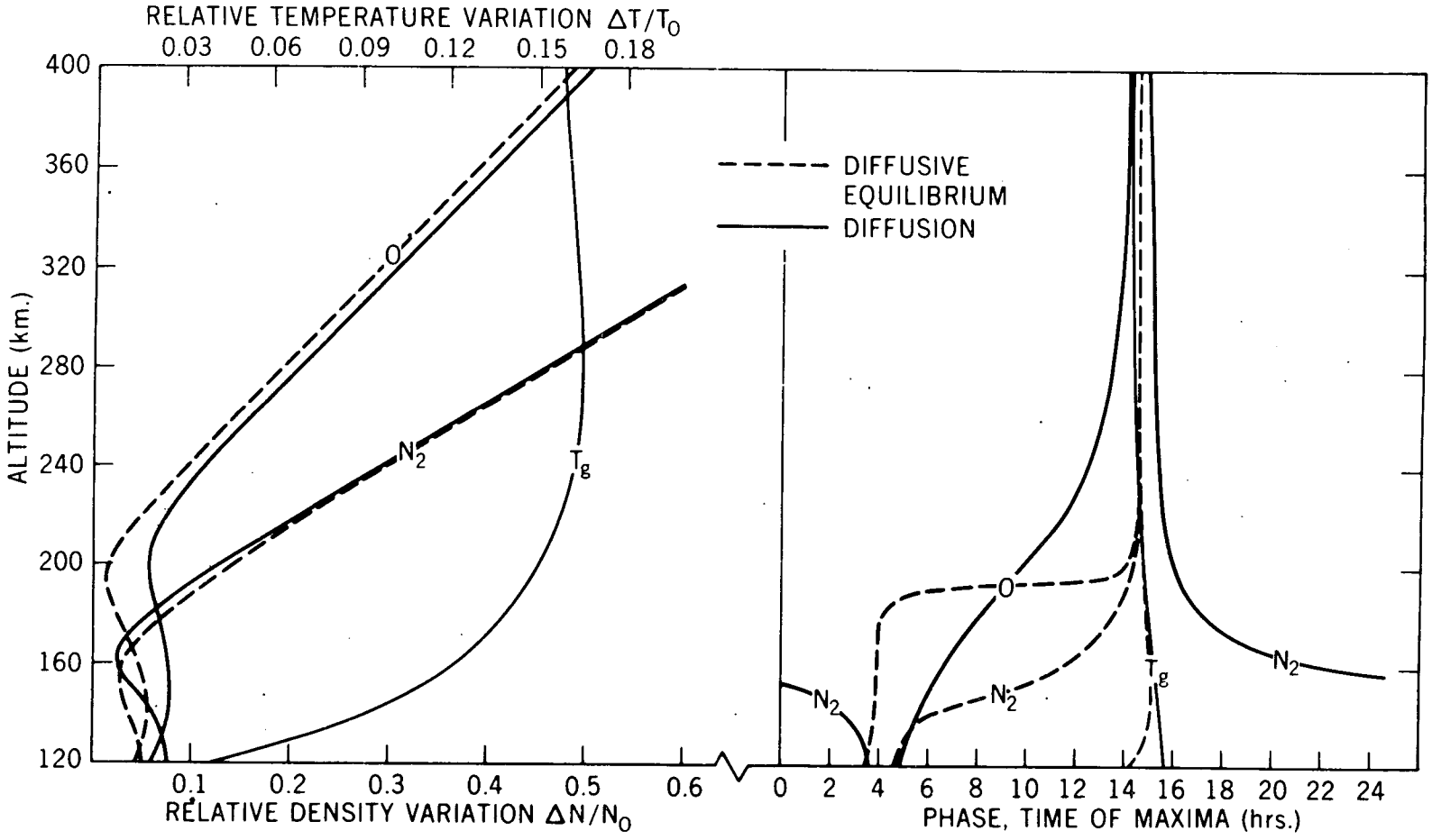

Figure 2. N_2-O-model. Relative amplitudes and phases for the O and N_2 concentrations and the gas temperature Tg. Dashed lines: diffusive equilibrium: solid lines: diffusion of O through N_2.

By treating diffusion and energy transport in a self consistent form our model is already too complex to lend itself readily to a discussion of the diffusion effect. We refer therefore to our earlier paper (Mayr and Volland, 1972a) in which we explicitly discussed the wind (diffusion) and temperature induced variations in atomic oxygen, thereby showing that it is essentially the global circulation in which O (or He) is removed in the late afternoon and supplied toward the morning hours thus shifting the density peak away from the temperature toward earlier local times.

In Figure 3 a composit picture for the diurnal component of the thermospheric temperature and composition, including N_2, O and He, is shown. Here the distributions for N_2, O and Tg (in solid line) were taken from the N_2-O-model described in Figure 2. In this N_2-O-model the diurnal variations in the mean molecular mass, Δm (t), corresponded to the diffusion of O through N_2, both being the major constituents up to 500 km. Our He-model of Figure 3 is characterized by diffusion of He through O and N_2, assuming that the mean molecular mass for O and N_2 combined does not vary as a function of time. That means: we set Δm from Figure 2 equal to zero.

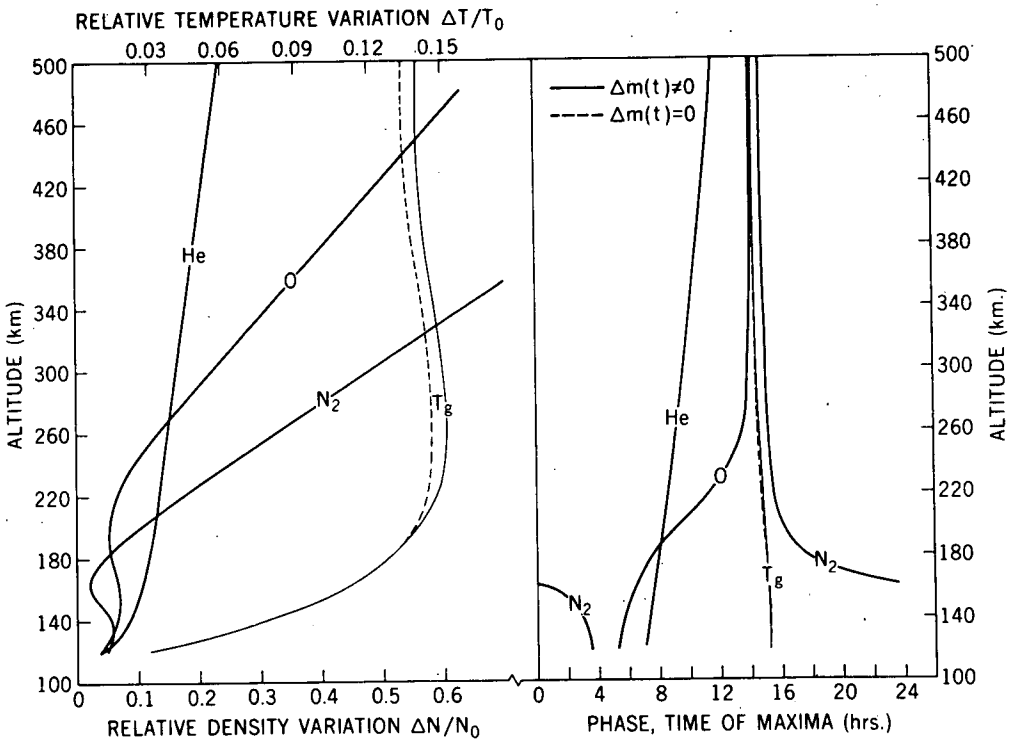

Figure 3. Relative amplitude and phases for N_2, O and He concentrations and gas temperature Tg. Dashed lines: diffusion of He through N_2 and O; solid lines: diffusion of O through N_2 (same as solid lines in Fig. 2).

Analogous to oxygen, He in diffusive equilibrium has an isopicnic level at about 280 km with its density being 12 hours out of phase with Tg below that level and in phase with the temperature above it. None of these characteristics are apparent in the "diffusion solution" that is shown in Figure 3. The He-phase, at lower altitudes in the morning hours, is advancing only gradually toward the temperature phase at higher altitudes with the density maximum still occurring in the late morning hours even at exospheric heights. Furthermore, the He amplitude, although somewhat leveling off at around 260 km, does not exhibit a minimum which would be characteristic for an isopicnic layer. Thus, as expected, He follows and emphasizes in every respect the trend that became apparent from the comparison between the "diffusive equilibrium" and the "diffusion" solutions in oxygen (Figure 2).

For the gas temperature the diffusion effect ($\Delta m \neq 0$) in the model of Figure 3 is the following: The diurnal redistribution in atomic oxygen, induced by the thermospheric circulation, produces a small depletion in the oxygen concentration during the late afternoon. Hence, in the upper thermosphere the total pressure amplitude is damped and with that the wind field and the adiabatic energy exchange, thus slightly increasing the temperature amplitude as shown from the comparison between the dashed and solid lines in Figure 3.

Radar temperature measurements (Mahajan, 1969) have suggested that a significant phase lag could exist between the exospheric temperature and density. Therefore a brief discussion of this problem is appropriate in the framework of our diffusion model.

Since we consider only the diurnal component, a comparison with observed temperature and density peaks would not be appropriate, in particular since it has been shown by Volland and Mayr (1972a) that the semi-diurnal component could play an important part in the temperature-density phase discrepancy. Our discussion should therefore be regarded as qualitative.

From Figure 3 it is apparent that in a diffusion model the phases in He and O precede that of Tg which in turn is followed by N_2. This particular configuration satisfies hydrostatic equilibrium which holds in spite of the fact that individual atmospheric constituents are not in diffusive equilibrium. In the upper thermosphere hydrostatic equilibrium implies that the total mass density is in phase with the temperature provided the temperature phase is height independent. With a height dependent temperature phase the variations in the total mass density will of course reflect to some degree the temperature phase in the underlaying atmosphere.

As the consequences of hydrostatic equilibrium the following characteristics can thus be inferred:

a) In the case illustrated in Figure 3, the phase of Tg is closest to that of O at 400 km where this constituent is dominant.

b) In spite of the relatively large phase difference between Tg and He up to 400 km (see Figure 3), He will be essentially in phase with the gas temperature close to 1400 LT at altitudes where this species dominates, and this is consistent with the satellite drag data which have so far not detected a significant height dependence in the phase of the total mass density.

c) To produce a significant phase discrepancy between the temperature and the total mass density, as suggested by the radar backscatter and satellite drag observations, it would be required that the temperature phase is height dependent occurring at lower altitudes at earlier local times.

The total mass density-temperature phase discrepancy is thus primarily determined by the energetics of the thermosphere which can be affected by a number of factors related to heat conduction, size of circulation or ion drag to name just a few.

To illustrate this point we show in Figure 4 three examples for the phase distributions of the temperature and the oxygen density. With dotted lines a case is illustrated in which the electron density is assumed to be infinitely large with the effect that the horizontal velocity is essentially forced to zero. In this example a onedimensional model is thus simulated. With this kind of a model heat conduction is by far the most important energy transport mechanism, and with the long heat conduction time in the lower thermosphere the temperature maximum develops there close to 1800 LT. However, at higher altitudes, where the heat conduction time decreases, the temperature maximum is shifted toward around 1600 LT with the consequence that at exospheric heights the phase of atomic oxygen, which in such a model is essentially in diffusive equilibrium, lags behind that of the gas temperature.

In the second example (dashed lines) a realistic N_e distribution is adopted where the horizontal velocities are significantly involved in the redistribution of energy and thus the characteristic time is sufficiently short so that the Tg phase occurs between 1400 and 1500 LT. In this particular case, O is not in diffusive equilibrium as demonstrated before, and consequently the oxygen phase preceeds that of Tg with a phase lag that shrinks to 15 min at 400 km.

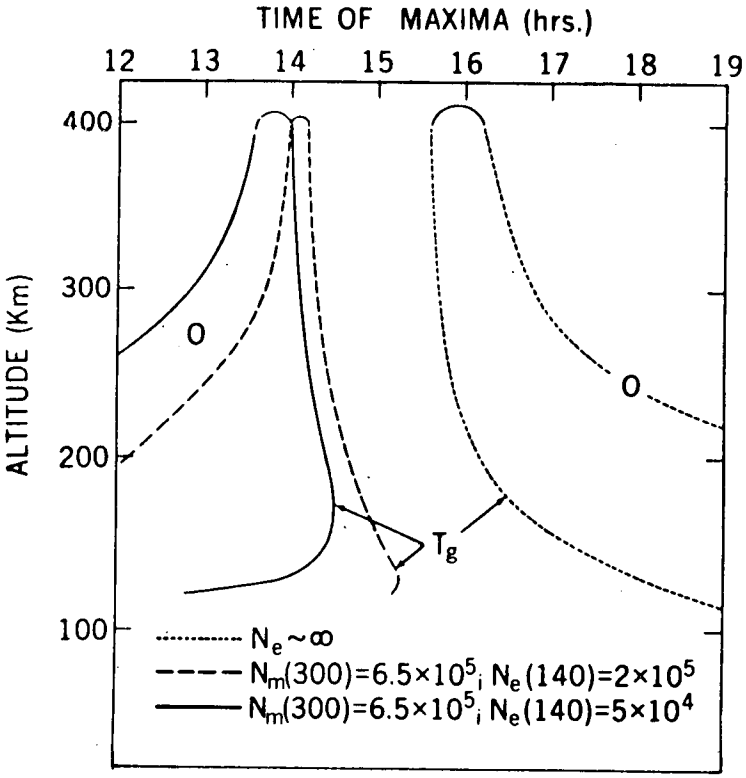

Figure 4. Phase relation for Tg and O-density for various electron density numbers.

This particular effect is further enhanced in the third example where we decrease the N_e density in the lower thermopshere from 2×10^5 to 4×10^4/cm without changing the F_2 maximum concentration N_m. In this case the horizontal velocities are primarily enhanced within the lower thermosphere with the effect that the response time of the gas temperature further decreases thus producing a Tg-phase close to 1300 LT at 120 km. The corresponding oxygen phase is now even further advanced to earlier local times, and it precedes that of the gas temperature by about half an hour at 400 km.

WIND FIELD

Figure 5 represents the amplitudes and phases of the horizontal, V, and the vertical, W, wind field from our N_2-O-model. With the solid lines, velocities of the N_2 component are shown, while the dashed line shows the horizontal velocity of O.

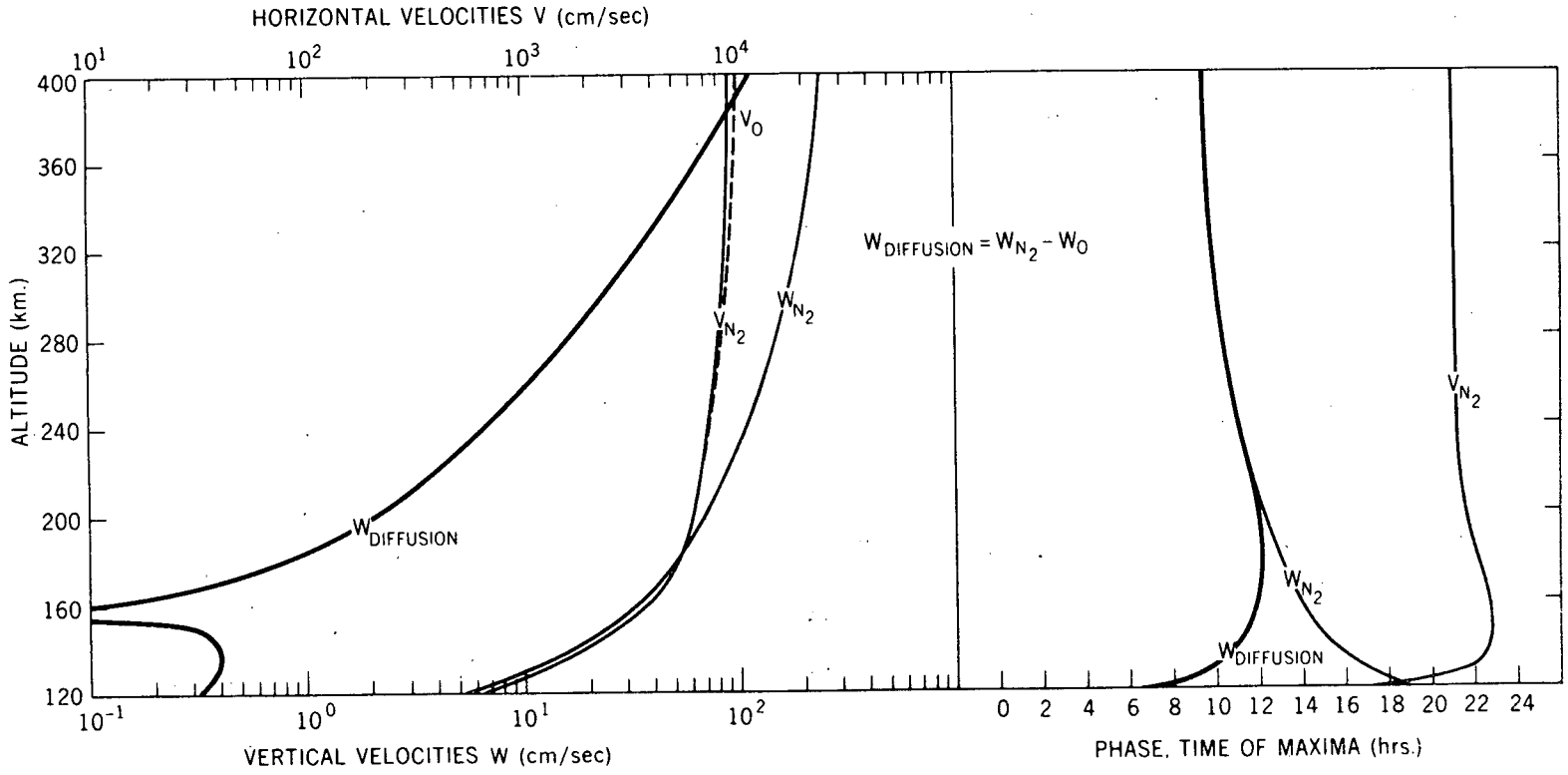

Figure 5. Amplitudes and phases for the velocity field from the N_2-O-diffusion model. V_i are the zonal velocities, W_i are the vertical velocities, $W_{N_2} - W_0$ is the vertical diffusion velocity.

It is apparent from this figure, that the horizontal transport velocities of both constituents are essentially identical up to exospheric heights where the difference is less than 10%, thus reflecting the dominance of the collisional momentum transfer in the horizontal momentum balance.

The height structure in both amplitude and phase of the thermospheric wind field is apparently very similar to our earlier results (Volland and Mayr, 1970) where the influence of viscosity has been considered in the form of an enhanced effective ion collision frequency.

Also shown in Figure 5 is the amplitude and phase of the diffusion velocity, $(W_{N_2} - W_2)$, which is over most of the altitude range about an order of magnitude lower than the vertical transport velocity. One can easily verify that the vertical pressure gradient of atomic oxygen increases in phase with the diffusive drag interaction. It is therefore apparent from the phase of the diffusion velocity that the diffusion process would tend to increase the oxygen density in the late morning hours, and basically the same mechanism is effective for He.

CONCLUSION

A two dimensional model of the thermosphere dynamics is presented in which the diffusion effects upon composition, temperature, and circulation are treated in a self consistent form. Two atmospheric components are considered for an O-N_2-model and for the diffusion of helium through the other constituents. It is concluded that the concept of diffusive equilibrium is not entirely justified in the diurnal tide. In particular it is shown that with respect to diffusive equilibrium:

1) The amplitudes of the helium and oxygen densities are significantly enhanced in the height regions where both constituents are minor, while the diffusion effects have little influence upon the temperature, total mass density and wind field.

2) The phases of the lighter and minor constituents are significantly advanced with respect to the gas temperature such that the maxima in the He and O concentrations occur e. g. at 300 km near 900 and 1300 LT respectively, a result which is basically in agreement with recent satellite observations (Hedin at al., 1972; Newton et al., 1972).

It has been shown in this paper that the phase delays between densities and temperature depend strongly upon the distribution of the electron density as it

affects the thermospheric circulation. Therefore the magnitude of this effect will certainly vary with the day to day fluctuations of the electron density and it should also be expected that it depends through the electron density on latitude, season and solar activity.

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REFERENCES

CIRA, COSPAR International Reference Atmosphere, 1965, North-Holland Publishing Comp., 1965.

Colegrove, F. D., Hanson, W. B., and Johnson, F. S.; Eddy Diffusion and Oxygen Transport in the lower Thermosphere, J. Geophys.Res. 70, 4931 - 4941, 1965.

Dalgarno, A.; Ambipolar Diffusion in the F-Region, Journ. Atm. Terr. Phys. 26, 939, 1964.

Dickinson, R. E., Lagos, C. P., and Newell, R. E.; Dynamics of the Neutral Thermosphere for small Rossby Number Motions, Journ. Geophys. Res. 73, 4299 - 4313, 1968,

Harris, I. and Priester, W.; Time Dependent Structure of the Upper Atmosphere, Journ. Atm. Sci. 19, 286 - 301, 1962.

Hedin, A. E., Mayr, H. G., Reber, C. A., Carignan, G. R., and Spencer, N. W.; A Global Empirical Model of Thermospheric Composition Based on OGO-6 Mass Spectrometer Measurements, Paper f. 45, COSPAR-Meeting, Madrid, 1972.

Jacchia, L. G.; Static Diffusion Models of the Upper Atmosphere with Empirical Temperature Profiles, Smithson. Contrib. Astrophys. 8, 215, 1965.

Lindzen, R. S. and Kuo, H L.; Reliable Method for the Numerical Integration of a Large Class of Ordinary and Partial Differential Equations, Monthl. Wea. Rev. 97, 732-734, 1969.

Mahaja, K. K.; Diurnal Variation of the Ion Temperature, Journ. Atm. Terr. Phys. 31, 93 - 101, 1969.

Mayr, H. G. and Volland, H.; Diffusion Model for the Phase Delay between Thermospheric Density and Temperature, J. Geophys. Res. 77, 2359 - 2367, 1972a.

Mayr M. G. and Volland, H.; Theretical Model for the Latitude Dependence of the Thermospheric Annual and Semiannual Variations, NASA-Document X-621-72-235, 1972b.

Newton, G. P., Pelz, D. T., and Kasprazak, W. T.; Composition of the Equatorial Thermosphere, EOS 53, 465, 1972.

Nicolet, M.; The Properties and Constitution of the Upper Atmosphere, in Ratcliffe (ed): "Physics of the Upper Atmosphere", Academic Press New York and London, 1960.

Volland, H. and Mayr, H. G.; A Theory of the Diurnal Variations of the Thermosphere, Ann. Geophys. 26, 907 - 919, 1970.

Volland, H. and Mayr, H. G.; A Numerical Study of a Threedimensional Spherical Thermospheric Density and Wind Model, NASA-Document X-621-72-234 1972.

Walker, R. E.; Measurements of O-O Diffusion Coefficient, Journ. Chem. Phys. 31, 2196 - 2197, 1961.